Assignment For class 12

Mathematics

Topic : Method to examine the consistency or inconsistency of a Non-Homogeneous equation

for a given matrix equation AX=B (difference between homogeneous & non-homogeneous discussed in video link provided by school)

- If $|A| \neq 0$, then the system is consistent and it has a unique solution given by $X=A^{-1}B$.
- If $|A| = 0$, & (adj A)B≠O(null matrix), then the system is inconsistent. **(note: Here we assume that *adj* A≠O,if adj A =O (Null Matrix),then the system may or may not be consistent)**
- If $|A| = 0$, & (adj A) B=O then the system may be consistent having infinitely many solutions or may be inconsistent.

Example 1 : Test for consistency / inconsistency of the given equation:

3x +11y =7, 6x +22y =5;

Solution :

$A=\begin{bmatrix} 3 & 11 \\ 6 & 22 \end{bmatrix}$, $X=\begin{bmatrix} x \\ y \end{bmatrix}$, and $B=\begin{bmatrix} 7 \\ 5 \end{bmatrix}$

$|A|= 66-66=0$, $adj\ A=\begin{bmatrix} 22 & -11 \\ -6 & 3 \end{bmatrix}$

$\therefore\ (adj\ A)\ B = \begin{bmatrix} 22 & -11 \\ -6 & 3 \end{bmatrix}\begin{bmatrix} 7 \\ 5 \end{bmatrix} = \begin{bmatrix} 99 \\ -27 \end{bmatrix}$

$\because$ $|A| = 0$, & $(adj\ A)\ B \neq O$ (null matrix), equations is inconsistent

Example 2 : Test for consistency / inconsistency of the equation:

4x +6y =7, 6x +9y =10.5;

solution: $A=\begin{bmatrix} 4 & 6 \\ 6 & 9 \end{bmatrix}$, $X=\begin{bmatrix} x \\ y \end{bmatrix}$, and $B=\begin{bmatrix} 7 \\ 10.5 \end{bmatrix}$

$|A|= 36-36=0$, $adj\ A=\begin{bmatrix} 9 & -6 \\ -6 & 4 \end{bmatrix}$

$\therefore\ (adj\ A)\ B = \begin{bmatrix} 9 & -6 \\ -6 & 4 \end{bmatrix}\begin{bmatrix} 7 \\ 10.5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = O$

$\because$ $|A| = 0$, & $(adj\ A)\ B = O$ (null matrix), equations is consistent with infinite solution (**solution to be discussed in the next assignment)**

Example 3. Test for consistency / inconsistency of the given equation

x +y - 2z =5, x-2y +z =-2 , -2x +y +z =-3:

$$A=\begin{bmatrix} 1 & 1 & -2 \\ 1 & -2 & 1 \\ -2 & 1 & 1 \end{bmatrix},\ X=\begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B=\begin{bmatrix} 5 \\ -2 \\ -3 \end{bmatrix}$$

$|A| = 1(-3) -1(3) -2(-3) = -3-3 +6=0$, and $adj\ A=\begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix}$

$$\text{Now ,}(adj\ A)\ B=\begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix}\begin{bmatrix} 5 \\ -2 \\ -3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}=O$$

Hence the given system of equations may be consistent with infinitely many solution or inconsistent. **(solution to be discussed in next assignmnt)**

Homework : Exercise 4.5. Q .2.ii),iii), iv) & Find λ so that the equations 3x -y +2z=1,2x +y +3z=0,

x-3y+ λ z=0 may have unique solution.